1.2

$$\frac{d^2h}{dt^2} = -g, \quad h(0)=5, \quad h'(0)=-3 \qquad h>0 \rightarrow \text{UPWARD}$$

$$\frac{dh}{dt} = -\int g \, dt + C_1$$

$$= -gt + C_1$$

$$h = \int (-gt + C_1)\, dt + C_2$$

$$h(t) = -\tfrac{1}{2}gt^2 + C_1 t + C_2$$ ← FAMILY OF SOLUTIONS OF ODE

$$h(0) = C_2 = 5$$

$$h'(0) = C_1 = -3$$

$$h(t) = -\tfrac{1}{2}gt^2 - 3t + 5$$ ← SOLUTION OF IVP

↖ PARTICULAR SOLUTION OF ODE $\frac{d^2h}{dt^2} = -g$

(IE. SOLUTION WITHOUT ARBITRARY CONSTANTS)

↑

COME OUT OF ANTIDIFFERENTIATION + SUBSEQUENT SIMPLIFICATION

EXISTENCE + UNIQUENESS THEOREM (E+U) FOR 1^{ST} ORDER IVPs

GIVEN THE IVP $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$

IF f AND f_y ARE BOTH CONTINUOUS IN AN OPEN REGION AROUND (x_0, y_0)

$\frac{\partial f}{\partial y}$

THEN THE IVP HAS A UNIQUE SOLUTION

ON AN OPEN INTERVAL AROUND x_0

(IE. THE SOLUTION IS CONTINUOUS + DIFFERENTIABLE FOR ALL x ~~IE~~ SUFFICIENTLY NEAR x_0

IE. $(x_0 - h, x_0 + h)$)

THAT OPEN INTERVAL IS CALLED THE INTERVAL OF DEFINITION

CONSIDER $\dfrac{dy}{dx} = \underbrace{2xy^2}$

$f(x,y)$ IS CONT. FOR ALL x, y IE. $\mathbb{R} \times \mathbb{R}$

$f_y = \dfrac{\partial f}{\partial y} = 2x \cdot 2y = 4xy$ IS CONT. ON $\mathbb{R} \times \mathbb{R}$

IE. ANY IVP OF THIS ODE
SATISFIES CONDITIONS OF E+U

IE. ANY IVP OF THIS ODE
HAS A UNIQUE SOLUTION

CLAIM: $y = \dfrac{1}{C - x^2}$ IS A FAMILY OF SOL'NS OF THIS ODE ✓

LHS: $\dfrac{dy}{dx} = \dfrac{d}{dx}(C-x^2)^{-1} = -1\,(C-x^2)^{-2}(-2x) = \dfrac{2x}{(C-x^2)^2}$

RHS: $2xy^2 = 2x \cdot \left(\dfrac{1}{C-x^2}\right)^2 = \dfrac{2x}{(C-x^2)^2}$

CLAIM: $y = 0$ IS ALSO A SOL'N ✓

$\dfrac{dy}{dx} = 0$

$2xy^2 = 2x \cdot 0^2 = 0$

THIS DOES NOT VIOLATE E+U
BECAUSE ONLY 1 OF THESE
WILL BE THE SOLUTION FOR ANY
SPECIFIC INITIAL VALUE

SINGULAR SOLUTION

$y = 0$ IS NOT PART OF THE FAMILY

$y = \dfrac{1}{C - x^2}$

NO VALUE OF C
WILL MAKE $\dfrac{1}{C - x^2} = 0$
AS FUNCTIONS

$\frac{dy}{dx} = 2xy^2, \quad y(0) = 100$

$\downarrow$

$y = \frac{1}{C - x^2}$ or $y = 0$

$\hookrightarrow y(0) = \frac{1}{C} = 100 \rightarrow C = \frac{1}{100}$

$y = \frac{1}{\frac{1}{100} - x^2} \cdot \frac{100}{100} = \frac{100}{1 - 100x^2}$ $\longleftarrow$ RATIONAL FUNCTION

CONT + DIFF ON ITS DOMAIN (MATH 1A TH'M)

INTERVAL OF DEFINITION (INCLUDES x_0 IN IVP)

$= \left(-\frac{1}{10}, \frac{1}{10}\right)$

DOMAIN OF $\frac{100}{1-100x^2}$

$1 - 100x^2 \neq 0$

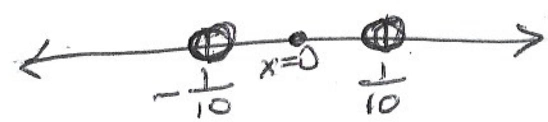
$1 \neq 100x^2$

$x^2 \neq \frac{1}{100}$

$x \neq \pm \frac{1}{10}$

f, f_y CONT ON $\mathbb{R} \times \mathbb{R}$

IE. $x \in (-\infty, \infty)$



$$\frac{dy}{dx} = 2xy^2 \ , \ y(0) = \frac{1}{100}$$

$$y = \frac{1}{100 - x^2}$$

INTERVAL OF DEF'N IS $(-10, 10)$